Instructions: Please write your solutions in LaTeX / Word or exquisite handwriting. Submission can be done individually or in pairs.

1. Let C be the projective curve defined by the equation $Y^2Z + YZ^2 = X^3 + XZ^2 + Z^3$ over $\mathbb{F}_2$. Note that this polynomial equation is the homogenization of the polynomial from problem 2 of the previous assignment. Feel free to use results you got there without reproving your claims.

    (a) Find all places of degree $1, 2, 3$ and 4 on C over $\mathbb{F}_2$. You may use a computer to speed things up.

    (b) Find the intersection divisors $C_{X=0} \cap C$, $C_{Y=0} \cap C$ and $C_{Z=0} \cap C$.

    (c) Find $\text{div}(1)$, $\text{div}(X/Z)$, $\text{div}(Y/Z)$, $\text{div}(X^2/Z^2)$ and $\text{div}(XY/Z^2)$.

    (d) Let P_∞ be the point on C at infinity. Find a basis for the Riemann-Roch spaces $\mathcal{L}(P_\infty)$, $\mathcal{L}(2P_\infty)$ and $\mathcal{L}(3P_\infty)$. You may use the fact that the genus of C is 1.

    (e) Find a basis for the Riemann-Roch space $\mathcal{L}(rP_\infty)$, for arbitrary nonnegative integer r.